Last time: Natural Deduction (more today)

Ex: Argue $(P \wedge Q) \wedge R \therefore S \vee (Q \vee T)$

Sol:

n	Statement	Justification
1	$(P \wedge Q) \wedge R$	Assumption
2	$P \wedge Q$	Conjunctive Simp. 1
3	$Q \wedge P$	Commutativity 2
4	Q	Conj. Simp. 3
5	$Q \vee T$	Disj Add 4
6	$(Q \vee T) \vee S$	Disj Add 5
7	$S \vee (Q \vee T)$	Comm 6

$A \wedge B \therefore A$

Ex: Argue $P \to Q,\ P \to R \therefore P \to (Q \wedge R)$.

Sol 1:

n	Statement	Justification
1	$P \to Q$	Assumption
2	$P \to R$	Assumption
3	$(P \to Q) \wedge (P \to R)$	Conj. Add. 1,2
4	$(\neg P) \vee Q) \wedge (P \to R)$	Mat. Imp. 3
5	$((\neg P) \vee Q) \wedge ((\neg P) \vee R)$	Mat Imp. 4
6	$(\neg P) \vee (Q \wedge R)$	Distribution 5
7	$P \to (Q \wedge R)$	Mat. Imp. 6

A better technique: Conditional Proof (Direct Proof)

IDEA: To prove $A \to B$:

Assume A, derive B via natural deduction.

Ex: Prove again $P \to Q, P \to R \therefore P \to (Q \wedge R)$.

Sol 2:

n	Statement	Justification
1	$P \to Q$	Assumption
2	$P \to R$	Assumption
3	P	Assumption for Cond. Proof.
4	Q	MP 1,3
5	R	MP 2,3
6	$Q \wedge R$	Conj. Add 4,5
7	$P \to (Q \wedge R)$	Cond. Proof 3-6. ∎

"Scoping bar" tells us where we can use new assumption

NB: This Conditional Proof / Direct Proof method can only be used to derive implications.

Argument form:

n	Statement	Justifications
⋮	⋮	⋮
i	A	Assumption for CP
⋮	⋮	⋮
j	B	
j+1	$A \to B$	CP i-j

→

Ex: Argue $P \to (Q \wedge R),\ R \to (P \wedge S) \therefore P \leftrightarrow R$.

$(P \to R) \wedge (R \to P)$

Sol:

n	Statements	Justifications
1	$P \to (Q \wedge R)$	Assumption
2	$R \to (P \wedge S)$	Assumption
3	$\vert\ P$	Assumption for CP (ACP)
4	$\vert\ Q \wedge R$	MP 1,3
5	$\vert\ R \wedge Q$	Com. 4
6	$\vert\ R$	Conj. Simp. 5
7	$P \to R$	CP 3-6
8	$\vert\ R$	ACP
9	$\vert\ P \wedge S$	MP 2,8
10	$\vert\ P$	Conj. Simp. 9
11	$R \to P$	CP 8-10
12	$(P \to R) \wedge (R \to P)$	Conj. Add. 7,11
13	$P \leftrightarrow R$	Biconditional Exp. 12 ☑

Proofs By Contradiction.

Argument Style:

n	Statement	Justifications
≀	≀	≀
i	¬B	Assumption for Contradiction (ARAA)
≀	≀	≀
j	$A \wedge (\neg A)$	Proof by Contradiction i-j (RAA)
j+1	$\neg\neg B$	

¬B

Ex: Argue $P \to R,\ Q \to S \ \therefore\ (P \vee Q) \to (R \vee S)$.

Sol:

n	Statement	Justifications
1	$P \to R$	Assumption
2	$Q \to S$	Assumption
→ 3	$P \vee Q$	ACP
4	$\neg(R \vee S)$	ARAA ←
5	$(\neg R) \wedge (\neg S)$	DM 4
6	$\neg R$	Conj. Simp. 5
7	$(\neg S) \wedge (\neg R)$	Comm 5
8	$\neg S$	Conj. Simp. 7
9	$\neg P$	MT 1, 6
10	$\neg Q$	MT 2, 8
11	$(\neg P) \wedge (\neg Q)$	Con. Adj. 9, 10
12	$\neg(P \vee Q)$ ↓ Contradiction.	DM 11
13	$(P \vee Q) \wedge (\neg(P \vee Q))$	Con. Adj. 3, 12
14	$\neg(\neg(R \vee S))$	RAA 4-13
→ 15	$R \vee S$	D.Neg. 14
16	$(P \vee Q) \to (R \vee S)$	CP 3-15

Next Time: Set Theory + Predicate Logic

Statement: ~~P:~~ $x + 5 = 7$ ← Not a statement in propositional logic...

$P(x) : x + 5 = 7$ ← predicate statement

Specialize: (something like $x = 3$), Quantify: (for every x, $x + 5 = 7$)